# Turbulent combined forced and free Convection Heat Transfer in vertical Tube Flow of Supercritical Fluids

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A theoretical analysis of combined forced and free convection heat transfer for turbulent flow is presented, with emphasis given to supercritical fluids. Free convection and other effects caused by the large property variations associated with heat transfer to supercritical fluids are accounted for in this surface renewal based formulation. Based on this analysis, the buoyancy force has been found to cause a deterioration in the heat transfer for up flow. This phenomenon is characterized by large increases in the mean periodicity of turbulence within the wall region. In addition to appropriately handling the effects of free convection for vertical forced convection of supercritical fluids, this analysis is also applicable to fluids with moderately varying properties.

## NOTATION

| | |
|---|---|
| $c_p$ | Specific heat at constant pressure |
| $f$ | Fanning friction factor |
| $f_r$ | Fanning friction factor for ribbed tube |
| $G$ | Mass velocity |
| $g$ | Acceleration due to gravity |
| $k$ | Thermal conductivity |
| $P$ | Pressure |
| $q''$ | Heat flux |
| $r$ | Radial coordinate |
| $T$ | Instantaneous temperature distribution |
| $T_i$ | Temperature at first instant of renewal |
| $u$ | Instantaneous velocity distribution |
| $U_i$ | Velocity at first instant of renewal |
| $v$ | Instantaneous velocity in $r$ or $y$ direction |
| $x$ | Axial coordinate |
| $y$ | Distance from wall |
| $\theta$ | Instantaneous contact time |
| $\phi$ | Contact time distribution function |
| $\rho$ | Density |
| $\tau$ | Mean residence time |
| $\mu$ | Viscosity |
| $\psi$ | General transport property |

### Subscripts

| | |
|---|---|
| b | Bulk stream conditions |
| c | Critical conditions |
| i | First instant of renewal |
| w | Wall conditions |

### Superscript

| | |
|---|---|
| — | Spatial mean conditions |

## INTRODUCTION

Several unusual heat transfer deterioration phenomena have been found to occur for turbulent convective heat transfer to supercritical fluids. Because of the important practical applications of these fluids, considerable study has been conducted over the past 15 years to provide a better understanding of these anomalies. Several mechanisms have been postulated in the literature for explaining the deteriorated regimes of heat transfer with supercritical fluids (**1**)–(**4**). Petukhov *et al.* (**3**) state that 'in spite of a large number of theoretical and experimental investigations, there is still not a clear understanding at present of the physical reasons for the local impairment of heat transfer at supercritical pressures, and there is no reliable engineering procedure for the calculation of heat transfer or the determination of the conditions under which heat transfer impairment occurs'. However, some progress has been made in the last few years in characterizing the different regimes of impaired heat transfer. Generally these have been classified into two groups called 'broad peaks' which occur over fairly long tube lengths, and 'sharp peaks' which are more localized. In contrast to 'broad peaks', the 'sharp peaks' are highly sensitive to mass velocity and heat flux. At the threshold limits small changes in either mass velocity or heat flux result in extremely large changes in wall temperature. Furthermore, the 'sharp peaks' referred to in this paper are dependent on the flow direction. In vertical tubes, 'broad peaks' occur for both upward and downward flow, whereas the 'sharp peaks' have only been observed for vertical up flow.

At least three different mechanisms have been suggested for broad temperature peaks (**2, 3, 5**). However, buoyancy forces do not appear to be a factor in the development of these peaks. The sharp peaks have been attributed to the effects of buoyancy forces (**1**). It is this phenomenon upon which attention is focused in the present study.

For laminar flow, free convection superimposed upon forced convection brings about an increase in heat transfer rate when the buoyancy force is in the flow direction.

Received 1 September 1979 and accepted for publication on 30 April 1980.

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The opposite effect for turbulent flow was first noticed in connection with the phenomenon of sharp wall temperature peaks with supercritical fluids (**6, 7**). Hall *et al.* (**1**) attributed these wall temperature peaks to a reduction in turbulence production as fluid in the wall region is accelerated by large buoyancy forces, followed by a recovery when the wall layer is moving faster than the fluid in the core.

A number of other studies with different fluids ranging from liquid metals (**8, 9**), low Reynolds number gas flow (**10**)–(**13**), and water (**6**), (**14**)–(**16**) have been published in recent years dealing with the subject of combined turbulent convection regime heat transfer in vertical tubes. The heat transfer deterioration effects of the buoyancy force in fluids with moderate property variations are not as pronounced as with supercritical fluids. Consequently, the importance of this phenomenon was not appreciated until it was brought into focus by the more severe wall temperature peaks in supercritical fluids.

In contrast to the findings alluded to above, previous published analyses for turbulent mixed convection (**17**)–(**19**) predict that buoyancy forces improve heat transfer in upflow. While experiments with gases by Bates *et al.* (**13**) confirm the buoyancy force effects observed with supercritical fluids, their numerical predictions using a modified van Driest mixing length model, which had been developed for pure forced convection, indicate that the effect of the buoyancy force is opposite to that found in experiments. Large variations in eddy diffusivity values and their dependence on heat flux (**9, 11, 20**) when buoyancy effects are significant, make it very difficult to apply classical turbulent heat transfer models to analyse mixed convection regime heat transfer.

A theoretical analysis of combined forced and free convection heat transfer for turbulent flow of supercritical fluids is presented in this paper which is based on the principle of surface renewal. This relatively new approach to the modelling of turbulent convection is based on the hypotheses that (1) an exchange of fluid between the turbulent core and the wall region occurs intermittently, and (2) unsteady molecular transport occurs during the brief residency of elements of fluid near the wall. This model was first adapted to turbulent convective momentum transport by Einstein and Li (**21**), and Hanratty (**22**), and has been subsequently utilized in the analysis of a broad range of turbulent transport processes (**23**)–(**25**), including heat transfer to supercritical fluids (**26**).

## ANALYSIS

Consideration will be first given to modelling the instantaneous transport to individual elements of fluid in residence near the wall, after which attention will be turned to the mean transport. The fluid to be studied is supercritical water.

### Instantaneous Transport

The instantaneous energy and momentum equations associated with the unsteady transport to individual elements of fluids in residence near the wall are written as

$$\rho c_p \frac{\partial T}{\partial \theta} = \frac{1}{r}\frac{\partial}{\partial r}\left(rk\frac{\partial T}{\partial r}\right) \tag{1}$$

and

$$\rho \frac{\partial u}{\partial \theta} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial u}{\partial r}\right) + (\rho_b - \rho)g \tag{2}$$

where $\theta$ is the instantaneous contact time and $U_i$ and $T_i$ are the velocity and temperature at the first instant of renewal. The initial and boundary conditions are $u(y, 0) = U_i$, $u(0, \theta) = 0$, $u(\infty, 0) = \text{finite}$, $T(y, 0) = T_i$, $T(0, \theta) = T_w$, and $T(\infty, \theta) = T_i$. In the high Reynolds number region of interest, the convective terms ($u\,\partial u/\partial x$, $v\,\partial u/\partial y$, $u\,\partial T/\partial x$, and $v\,\partial T/\partial y$) and the axial pressure gradient term ($\partial P/\partial x$) have been found to be secondary (**28**) and are therefore excluded. Hall (**5**) has shown that the acceleration of supercritical fluids, caused by expansion as the fluid is heated, is not important for conditions with significant buoyancy influence. Free convection and other effects caused by the large property variations associated with heat transfer to supercritical fluids are accounted for in this formulation. The thermophysical properties are specified in accordance with the information shown in Fig. 1 for water.

In regard to the initial and boundary conditions, because molecular penetration is shallow for high Peclet number conditions, $U_i$ and $T_i$ are approximated by $U_b$ and $T_b$, respectively. This type of assumption for $U_i$ and $T_i$ has been found to be useful in previous surface renewal based analyses (**26**).

### Mean Transport

The contribution of the instantaneous transport within the numerous elements of fluid at the surface to the spatial (or time) mean transport is obtained by the use of

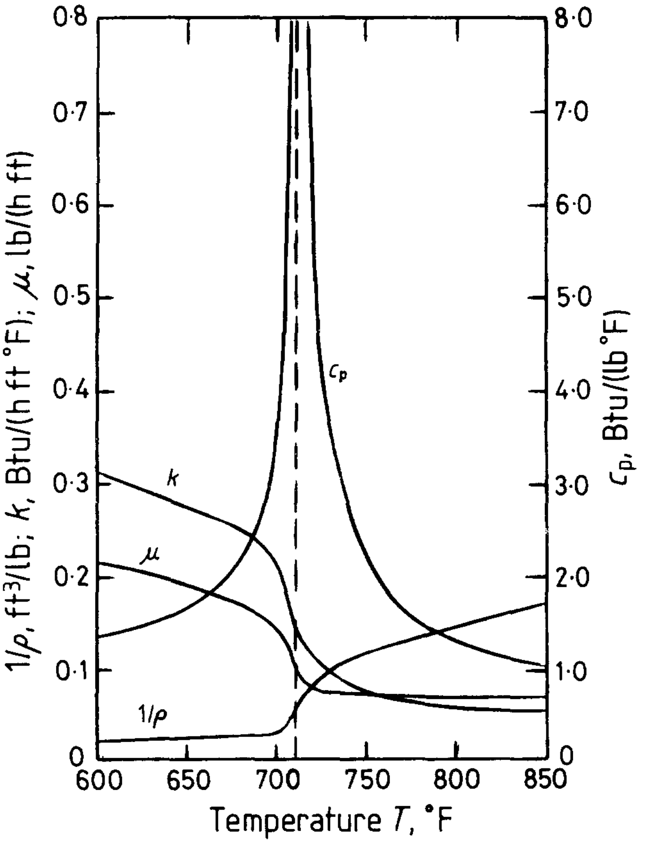

Fig. 1. Thermophysical properties of water at 3300 lb/in² abs; $T_c = 711°$ F. (From Swenson *et al.* (**27**))

the statistical age distribution concept. Based on this concept, the mean transport property $\bar{\psi}$ (velocity or temperature profiles, wall shear stress, or heat flux) is expressed in terms of the instantaneous property $\psi(\theta)$ by an expression of the form

$$\bar{\psi} = \int_0^\infty \psi(\theta)\phi(\theta)\,\mathrm{d}\theta \qquad (3)$$

The contact time distribution function $\phi(\theta)$ is defined such that the product $\phi(\theta)\,\mathrm{d}\theta$ represents the fraction of the surface with contact time between $\theta$ and $\theta + \mathrm{d}\theta$. Because predictions for $\bar{\psi}$ have been found to be fairly insensitive to the form of $\phi(\theta)$ selected, the convenient uniform distribution proposed by Higbee (**29**)

$$\phi(\theta) = \frac{1}{\tau}U(\theta - \tau) \qquad (4)$$

will be used. (Other distributions such as Danckwerts (**30**) exponential contact time distribution are sometimes used.)

The mean residence time $\tau$ will be seen to characterize the turbulent flow in the vicinity of the deteriorated heat transfer. Therefore, emphasis will be placed on the theoretical prediction of this property. This will be accomplished by first solving (numerically) the above systems of equations for the instantaneous and mean transport properties $\psi(\theta)$ and $\bar{\psi}$. $\tau$ will then be expressed in terms of the mean wall shear stress $\bar{\tau}_w$, which will be approximated.

### Solutions

The momentum and energy eqs. (1) and (2) are nonlinear because of the variations in properties with temperature. These nonlinear partial differential equations were solved simultaneously on an IBM 370-155 computer using an explicit finite difference numerical method. The approximations suggested by Carnahan *et al.* (**31**) were used in representing the different nonlinear terms in the equations in the finite difference form. Properties over a single time-step were taken as constant by assigning them values at the beginning of the time-step. Because of the small time-step used in the solution for reasons of stability, iteration across a time-step was found to be unnecessary. The accuracy of the numerical solution was established by comparing its predictions with the results obtained from the analytical solution of a constant property problem.

Predictions for the mean residence time $\tau$ can be obtained from the model by utilizing empirical information pertaining to the mean wall shear stress. Alternately, if experimental values are available for $\tau$, they can be used to predict the mean wall shear stress, as well as the velocity and temperature profiles and Nusselt numbers. Experimental values for $\tau$ have been obtained for fluids with constant properties (**32**) which were found to be in good agreement with the theoretical predictions. But no such data are presently available for turbulent flow of supercritical fluids.

Very little experimental information is available for determining the mean wall shear stress with heat transfer to variable property fluids. In fact, there are no experimental data for either mean wall shear stress or wall layer mean velocity profiles in the pseudo-critical temperature region. However, in the previous application of the surface renewal model for supercritical fluids (**26**), the authors found that the use of a mean wall shear stress based on Allen and Eckert's (**33**) formulation for the friction factor led to quite good predictions for heat transfer. The same approximation is used in the present study.

## RESULTS AND DISCUSSION

In a previous analysis of turbulent convective heat transfer to supercritical fluids (**26**), increases in the coefficient of heat transfer were found to accompany decreases in predictions for $\tau$, and vice versa. Of course, for a uniform wall heat flux boundary condition, decreases in the coefficient of convective heat transfer are associated with increases in $T_w$. Therefore, the results of this study pertaining to the deterioration in heat transfer are presented in terms of the mean period of surface renewal (or mean burst period) which characterizes the wall layer turbulence as well as the thermal properties such as the coefficient of heat transfer and $T_w$.

Figure 2 illustrates the influence of the buoyancy force on predictions for turbulent forced convection in vertical tube flow. Curves 1 and 2 represent predictions for up flow, with the buoyancy term omitted for Case 1 and included for Case 2. Curve 3 is for down flow with the buoyancy term included. In each of these three cases, predictions for $\tau$ are identical for the situation in which no heating occurs ($T_w = 620°$ F). As $T_w$ increases above this value, a gradual bifurcation is seen to occur in the predictions for $\tau$ for these three cases. This divergence in the predictions for $\tau$ for cases 2 and 3 from case 1 can be attributed totally to the influence of buoyancy. For case 1 in which the free convection effects are not modelled, the decrease in $\tau$ occurs because of the large variations in density and viscosity in the wall layer. For case 2, the initial drop in predictions for $\tau$ with increases in $T_w$ is caused by property variation effects apart from free

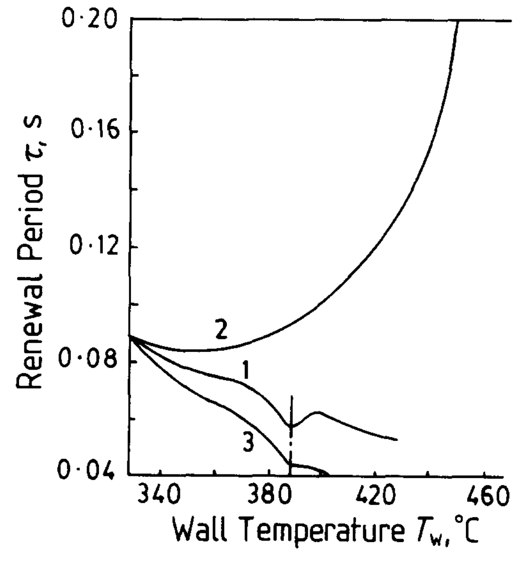

Fig. 2. Characterization of buoyancy force effect on mixed convection turbulent flow heat transfer. Water: $G = 407$ kg/(m$^2$ s); $D = 24{\cdot}38$ mm; $P = 24{\cdot}82 \times 10^{-6}$ N/m$^2$; $T_b = 327°$ C; $T_c = 388°$ C. 1, No buoyancy force; 2, Buoyancy force in flow direction; 3, Buoyancy force opposite to flow direction

convection. However, the buoyancy effects gradually bring about a minimization in $\tau$ as $T_w$ approaches $T_c$, after which larger and larger increases in $\tau$ are predicted for increasing $T_w$. In the region $T_w > T_c$, the dominance of the buoyancy force, as reflected in the large values of $\tau$, increases with $T_w$ until a severe deterioration in heat transfer occurs. The sharp increase in $\tau$ immediately preceding the localized deteriorated heat transfer can be interpreted as a stabilization of the wall layer which is caused by buoyancy forces in the direction of flow. This result is in qualitative agreement with the reduction in turbulence and thickening of the viscous sublayer experimentally observed for supercritical carbon dioxide (**34**) and gases (**12**) in the region of deteriorated heat transfer. Reduced wall layer turbulence and the associated poor heat transfer predicted by the model gives rise to sharp peaks in the wall temperature for a uniform flux boundary condition.

For case 3 in which the buoyancy forces oppose the inertial forces, the free convection is seen to bring about an increase in the wall region turbulence. Consequently, a general enhancement in heat transfer is realized in this situation, as contrasted to the severe deterioration with up flow. This conclusion is in agreement with the trends in experimental data reported by Bourke (**34**) and Jackson (**7**) for supercritical carbon dioxide, Shitsman (**6**) for water with temperatures below $T_c$, and Khosla (**20**) for gases. Incidentally, the dip in predictions for $\tau$ in the vicinity of the pseudo-critical temperature accompanies a localized enhanced heat transfer which also has been observed (**27**, **35**) and predicted (**26**) for down flow at low heat fluxes in this temperature region.

Figure 3 shows the effects of mass velocity and $T_w$ on predictions for $\tau$ for up flow with bulk fluid temperature below $T_c$ and $T_w$ below, equal to, and above $T_c$. Although the property variations for these three temperature conditions differ greatly, the mass velocities at the threshold limits for deteriorated heat transfer are of the same order of magnitude. This demonstrates the dominant influence of mass velocity in determining the region of free convection induced deterioration in the heat transfer. Similar domination by the mass velocity has been predicted by various dimensionless number criteria (**36**). It should be noted that the mass velocity limit for the three values of $T_w$ considered are in good agreement with experimental data (**16**, **37**). It is of particular interest to note that both theory and experiment (**6**, **15**, **16**) indicate that the heat transfer deterioration phenomenon occurs at wall and fluid temperatures below $T_c$.

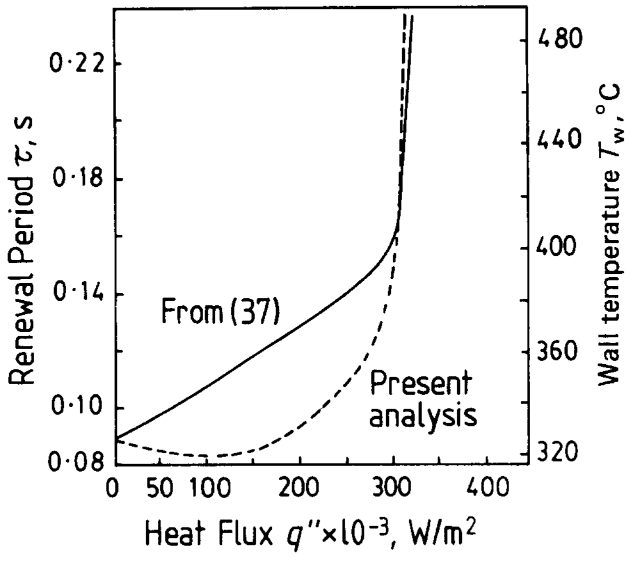

Fig. 4. Comparison of heat flux on renewal period and wall temperature

The effect of heat flux on wall temperature and on predictions for $\tau$ are compared in Fig. 4. The experimental data for $T_w$ indicate that a sharp peak occurs in the vicinity of $10^5$ Btu/(h ft$^2$). The prediction of buoyancy induced stabilization (i.e., very large $\tau$) is in quite good agreement with these data.

The effect of diameter on predictions for $\tau$ for up flow is shown by Fig. 5. The implied prediction of deteriorated

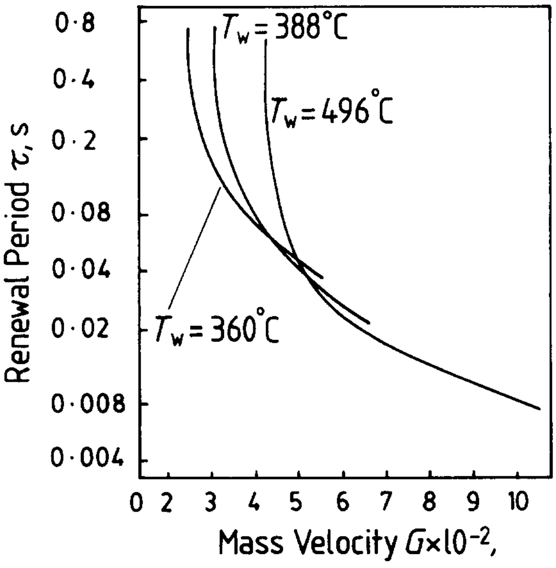

Fig. 3. Mass velocity effect on renewal period

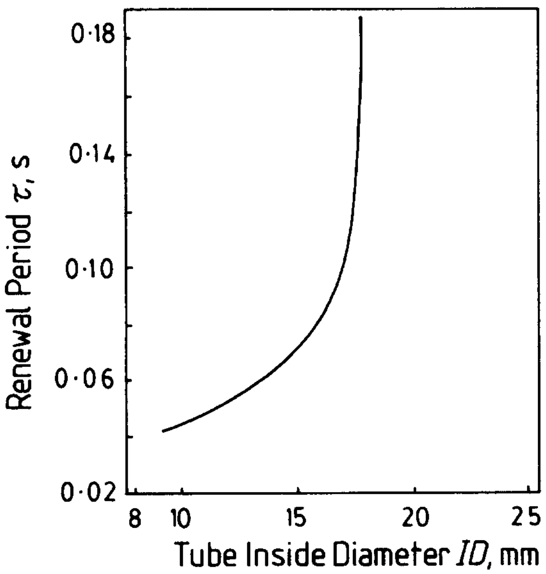

Fig. 5. Tube diameter effect on renewal period

heat transfer is in reasonably good agreement with the data of Ackerman (**37**).

Calculations have also been made for turbulent up flow of a supercritical fluid in a ribbed tube (**36**). This type of tube has recently been shown by Ackerman (**37**) to suppress the development of wall temperature peaks. Specifically, for the same tube diameter and mass velocity for which smooth tubes exhibited a deterioration in heat transfer, no temperature peaks were found in ribbed tubes, even at much higher heat fluxes. Based on measurements for the isothermal friction factor for the ribbed vs smooth tube reported by Ackerman ($1{\cdot}25 = f_r/f$), the present analysis indicates that no peaks in wall temperature would be experienced for the test conditions reported.

Predictions for threshold limits on the conditions under which buoyancy forces superimposed on forced convection become significant have been previously obtained on the basis of order of magnitude analyses using dimensionless numbers. Various criteria have been published in the recent literature (**36**). A dimensionless number criterion also has been obtained using the surface renewal model (**36**). The momentum equation was solved using constant properties and a constant buoyancy force term. The relationship for $\tau$ obtained from the solution of this equation was used to set the criterion for significant buoyancy force. This criterion is in good agreement with the limit criterion obtained by Hall (**5**) using an entirely different simplified 'two-region' model.

It should be noted that the predicted stabilizing influence of buoyancy on turbulent up flow is similar to the effect of favourable pressure gradients in highly accelerated flow or transitional turbulent flow. Previous surface renewal based analyses of these processes (**28, 36**) indicate that the pressure gradient term in the momentum equation contributes greatly to the stabilization of the flow and the eventual relaminarization. In general, forces which act in the direction of flow, such as buoyancy or pressure, appear to neutralize the effects of shearing forces in producing turbulence.

Because the primary emphasis of this study pertains to the region of deteriorated heat transfer which occurs well downstream from the thermal entrance, the convective terms ($u\ \partial u/\partial x$, $v\ \partial u/\partial y$, $u\ \partial T/\partial x$ and $v\ \partial T/\partial y$) which are significant in this region, were excluded in the underlying momentum and energy equations. Evidently the thermal boundary layer does not extend far enough into the fluid to produce significant buoyancy forces in this rather long thermal entrance region. In order to develop the capability of predicting the axial location at which the buoyancy-induced sharp peak in wall temperature occurs, a non-local surface renewal based analysis which includes these convective terms could be formulated. In this regard, an analysis of the thermal entrance region problem for constant property tube flow has recently been developed on the basis of the surface renewal principle (**36**).

In regard to the abrupt elimination of the deterioration in heat transfer which is characteristic of this phenomenon, the present analysis gives rise to predictions of buoyancy-induced velocity peaking in the wall layer region at the axial location of the wall temperature peaks (**36**). This finding is compatible with Hall's (**1**) hypothesis that turbulence production is increased after being minimized, as a consequence of the wall layer overtaking the core (i.e., negative shear stress in the core).

On the basis of their experimental data with supercritical carbon dioxide, Bourke and Pulling (**34**) postulate that the end of heat transfer deterioration is caused by the velocity increases due to fluid expansion as it is heated through the critical temperature. However, velocity peaks near the wall layer could not be seen in their data because of lack of measurements in the vicinity of the wall layer. The present analysis, which indicates the existence of a peak velocity in the vicinity of the wall layer, reinforces the statement by Kenning *et al.* (**16**) that further velocity profile measurements are needed in the wall layer region at the locations of the temperature peaks.

Incidentally, the present analysis indicates that heating through the critical temperature is not needed to cause large increases in wall layer velocities. Buoyancy forces have been found to cause substantial velocity increases in the wall layer for liquid metals (**9**) and air (**11**).

## CONCLUDING REMARKS

In this study, the effects of free convection on turbulent forced convection heat transfer in vertical tubes has been analysed, with emphasis given to supercritical fluids. Whereas previous classical analyses of this complex problem have not been successful, the present surface renewal based formulation leads to predictions for the mean transport properties which are consistent with experimental observations. Based on this analysis, the buoyancy force is seen to cause a deterioration in the heat transfer for up flow. This deterioration phenomenon is characterized by large increases in $\tau$, which is consistent with experimental observations pertaining to turbulence activity near the wall. In contrast, the analysis indicates that the buoyancy force intensifies the turbulence within the wall region for down flow, thus leading to improved heat transfer. This result also is consistent with experimental data.

In addition to appropriately handling the effects of free convection for vertical forced convection of supercritical fluids, this analysis is also applicable to fluids with moderately varying properties. It should be noted that Hall found that his simple two-region model was inadequate for these fluids.

To date, design criteria for preventing the sharp temperature peaks in nuclear reactors and fossil fired steam generators have been dependent upon order of magnitude considerations and empirical data. The present analysis provides a theoretical basis for understanding the true nature of the underlying mechanism associated with deteriorated heat transfer. This theoretical foundation also provides a means by which more reliable limit criteria can be established.

In order to augment the theoretical findings of this study, it is clear that additional experimental measurements pertaining to the role of free convection in turbulent forced convection heat transfer are needed. Measurements of the mean wall shear stress and the velocity profile very near the wall in the vicinity of the wall temperature peak would be most useful. Of course, such measurements are not easily made under circumstances in which great property variations occur. As an alternative, measurements of the mean burst period τ, which are more easily obtained, are suggested. The coupling of such measurements with the theoretical model developed herein would provide the basis for a better understanding of this complex problem.

## REFERENCES

(1) HALL, W. B., JACKSON, J. D., and WATSON, A. *Proc. Instn mech. Engrs* 1968, **182**, Part 31, 10

(2) SHIRALKAR, B. and GRIFFITH, P. *J. Heat Transfer, Trans. ASME* 1970, **92**, 465

(3) PETUKHOV, B. S., PROTOPOPOV, V. S., and SILIN, V. A. *Int. Chem. Eng.* 1973, **13**, 235

(4) TSUNGE, A., TANAKA, H., HIRATA, M., and NISHIWAKI, N. *Fifth Int. Heat Transfer Conference*, Round Table Discussion, 1974, RT 8-2, Vol. VII

(5) HALL, W. B. *Advances in Heat Transfer*, 1971, Vol. 7, 1 (Academic Press)

(6) SHITSMAN, M. E. *Proc. Instn mech. Engrs* 1968, **182**, Part 31, 36

(7) JACKSON, J. C. and EVANS-LUTTERODT, K. Report N.E. 2, Simon Engineering Labs, University of Manchester, 1968.

(8) BUHR, H. O., CARR, A. D., and BALZHISER, R. E. *Int. J. Heat Mass Transfer* 1968, **11**, 641

(9) BUHR, H. O., HORSTEN, E. A., and CARR, A. D., National Heat Transfer Conference, 1972, ASME 72-HT-21

(10) STEINER, A., *J. Fluid Mech.* 1971, **47**, 503

(11) CARR, A. D., CONNOR, M. A., and BUHR, H. O. National Heat Transfer Conference 1972, ASME 72-HT-19

(12) BYRNE, J. E. and EJIOGU, E. *Symposium on Heat and Mass Transfer by Combined Forced and Natural Convection* 1972, 40 (I. Mech. E., London)

(13) BATES, J. A., SCHUNALL, R. A., HASEN, G. A., and McELIGOT, D. M. *Fifth Int. Heat Transfer Conference* 1974, Vol. II, 141

(14) VIKHREV, Y. V., BARULIN, Y. D., and KON'KOV, A. S. *Teploenergetika* 1967, **14** (9), 80

(15) ALFEROV, N. S., RYBIN, R. A., and BALUNOV, B. F., *Teploenergetika* 1969, (12), 66

(16) KENNING, D. B. R., SHOCK, R. A. W., and POON, J. Y. M. *Fifth Int. Heat Transfer Conference* 1974, Vol. III, 139

(17) OJALVO, M. S., ANAND, D. K., and DUNBAR, R. P. *J. Heat Transfer, Trans. ASME* 1967, **89**, 328

(18) SHERWIN, K. *Brit. Chem. Eng.* 1971, **16**, 367

(19) TANAKA, H., TSUGE, A., HIRATA, M., and NISHIWAKI, N. *Int. J. Heat Mass Transfer* 1973, **16**, 1267

(20) KHOSLA, J., HOFFMAN, T. W., and POLLOCK, K. G. *Fifth Int. Heat Transfer Conference* 1974, Vol. III, 144

(21) EINSTEIN, H. A. and LI, H., *ASCE, J. Eng. Mech. Div.* 1956, **82**, 293

(22) HANRATTY, T. J., *AIChE J.* 1956, **2**, 359

(23) THOMAS, L. C. and WOOD, M. L. *Int. J. Heat & Fluid Flow* 1979, **1** (2), 93

(24) THOMAS, L. C. *J. Heat Transfer, Trans. ASME* 1970, **92**, 565

(25) MEEK, R. L. and BAER, A. D. *AIChE J* 1970, **16**, 1100

(26) KAKARALA, C. R. and THOMAS, L. C. *Fifth Int. Heat Transfer Conference*, 1974, Vol. II, 45

(27) SWENSON, H. S., CARVER, J. R., and KAKARALA, C. R. *J. Heat Transfer, Trans. ASME* 1965, **87**, 477

(28) GROSS, R. J. and THOMAS, L. C. *J. Heat Transfer, Trans. ASME* 1972, **94**, 494

(29) HIGBEE, R. *Trans. AIChE* 1935, **31**, 65

(30) DANCKWERTS, P. V., *Ind. Engng. Chem.* 1951, **43**, 1460

(31) CARNAHAN, B., LUTHER, H. A., and WILKES, J. O., *Applied Numerical Methods* 1969 (John Wiley & Sons, Inc.)

(32) THOMAS, L. C. and GREENE, H. L., Symposium on Turbulence in Liquids, The University of Missouri, Rolla, 1973

(33) ALLEN, R. W. and ECKERT, E. R. G. *J. Heat Transfer, Trans. ASME* 1964, **86**, 301

(34) BOURKE, P. J. and PULLING, D. J. National Heat Transfer Conference 1971, ASME 71-HT-24

(35) HALL, W. B., JACKSON, J. D., and KHAN, S. A. *Third Int. Heat Transfer Conference*, 1966

(36) KAKARALA, C. R. PhD Thesis, University of Akron, 1976.

(37) ACKERMAN, J. W. *J. Heat Transfer, Trans. ASME* 1970, **92**, 490